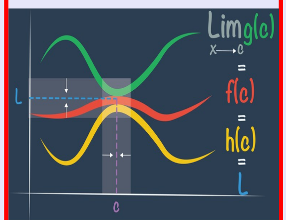
Math 261

Spring 2022

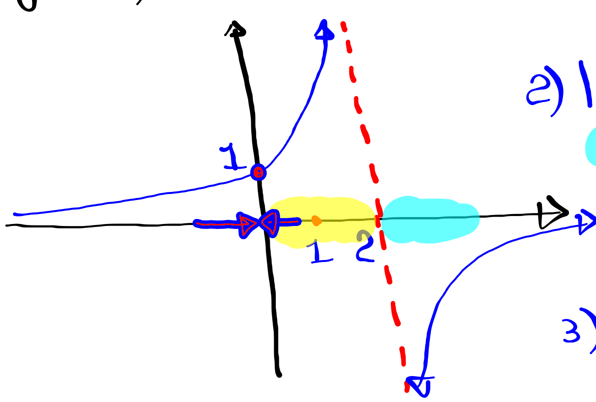
Lecture 3



Class QZ 2

Consider the graph of

$y = f(x)$ below



1) $\lim_{x \to 2^-} f(x) = \boxed{\infty}$ ✓

2) $\lim_{x \to 2^+} f(x) = \boxed{-\infty}$ ✓

3) $\lim_{x \to 0} f(x) = \boxed{1}$ ✓

If $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$, then

1) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L_1 + L_2$

2) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L_1 - L_2$

3) $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L_1 \cdot L_2$

4) $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L_1}{L_2}$ if $L_2 \neq 0$

5) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$ when n is even, then $L_1 \geq 0$

6) $\lim_{x \to a} K = K$

7) $\lim_{x \to a} K f(x) = K \lim_{x \to a} f(x) = K L_1$

8) $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n = (L_1)^n$

Evaluate $\lim_{x \to \frac{1}{3}} 12x = 12 \lim_{x \to \frac{1}{3}} x$

$= 12 \cdot \frac{1}{3} = \boxed{4}$

Evaluate $\lim_{x \to -\sqrt{5}} (-5x^2) = -5 \lim_{x \to -\sqrt{5}} x^2 = -5 \left[\lim_{x \to -\sqrt{5}} x\right]^2$

$= -5[-\sqrt{5}]^2 = -5 \cdot (+5) = \boxed{-25}$

use rule of limits to evaluate

$$\lim_{y \to -1} (y^6 - 12y - 12)$$

$$= \lim_{y \to -1} y^6 - \lim_{y \to -1} 12y - \lim_{y \to -1} 12$$

$$= \left[\lim_{y \to -1} y\right]^6 - 12 \lim_{y \to -1} y - \lim_{y \to -1} 12$$

$$= (-1)^6 - 12(-1) - 12 = 1 \cancel{+12} \cancel{-12} = \boxed{1}$$

Find $\lim_{x \to 3} \dfrac{x^2 - 2x}{x + 1} = \dfrac{\lim_{x \to 3} (x^2 - 2x)}{\lim_{x \to 3} (x + 1)}$

$$= \frac{3^2 - 2(3)}{3 + 1} = \frac{9 - 6}{4} = \boxed{\frac{3}{4}}$$

Find $\lim_{x \to 1} \dfrac{x^4 - 1}{x - 1} = \dfrac{1^4 - 1}{1 - 1} = \dfrac{0}{0}$ I.F.

$$\lim_{x \to 1} \frac{(x^2 - 1)(x^2 + 1)}{x - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{x-1}}$$

$$= \lim_{x \to 1} \left[(x+1)(x^2+1)\right]$$

$$= (1 + 1)(1^2 + 1) = 2(2)$$

$$= \boxed{4}$$

Graph $f(x)=\frac{x}{x-2}$, then evaluate

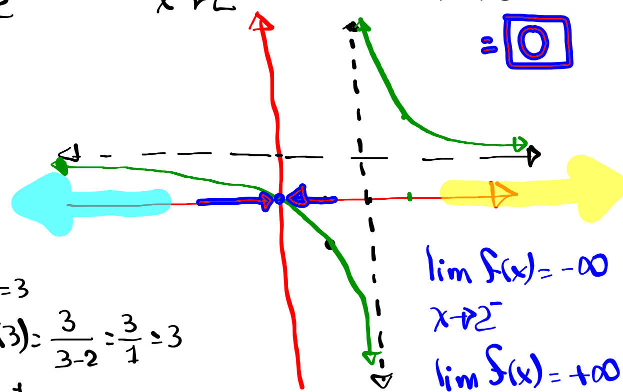
$\lim_{x\to2^-} f(x)$, $\lim_{x\to2^+} f(x)$, $\lim_{x\to2} f(x)$, and $\lim_{x\to0} f(x)$ $=\boxed{0}$

$f(x)=\frac{x}{x-2}$

$x-2\neq0$

$x\neq2$

V.A.



$x=3$

$f(3)=\frac{3}{3-2}=\frac{3}{1}=3$

$x=1$

$f(1)=\frac{1}{1-2}=-1$

$\lim_{x\to2^-} f(x)=-\infty$

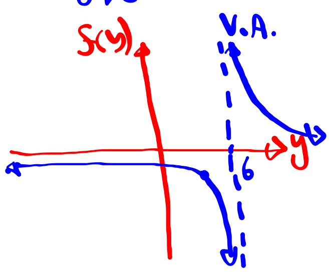
$\lim_{x\to2^+} f(x)=+\infty$

$\lim_{x\to2} f(x)=$ D.N.E.

$\lim_{x\to+\infty} f(x)=\boxed{1}$ $\lim_{x\to-\infty} f(x)=\boxed{1}$

Evaluate:

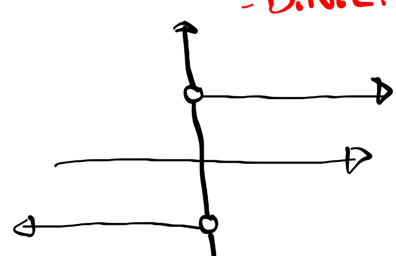
1) $\lim_{y\to6}\frac{y+6}{y^2-36}=\lim_{y\to6}\frac{\cancel{y+6}}{(\cancel{y+6})(y-6)}$



$=\lim_{y\to6}\frac{1}{y-6}$

$=\boxed{\text{D.N.E.}}$

3) $\lim_{x\to0}\frac{|x|}{x}$ = D.N.E.



$\lim_{x\to0^+}\frac{|x|}{x}=1$, $\lim_{x\to0^-}\frac{|x|}{x}=-1$

2) $\lim_{y\to9}\frac{y-9}{\sqrt{y}-3}=\frac{0}{0}$

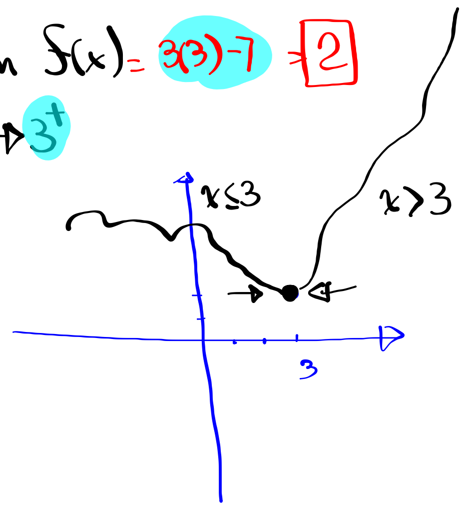
$\lim_{y\to9}\frac{\cancel{(y-9)}(\sqrt{y}+3)}{\cancel{(\sqrt{y}-3)}(\sqrt{y}+3)}=\lim_{y\to9}(\sqrt{y}+3)=\boxed{6}$

$$f(x) = \begin{cases} x-1 & \text{if } x \le 3 \\ 3x-7 & \text{if } x > 3 \end{cases}$$

Find

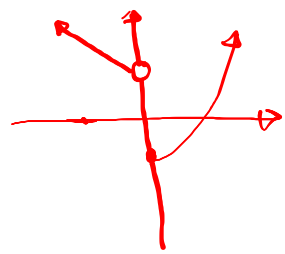
$$\lim_{x \to 3^-} f(x) = (3-1) = \boxed{2}$$

$$\lim_{x \to 3^+} f(x) = 3(3)-7 = \boxed{2}$$



$$\lim_{x \to 3} f(x) = \boxed{2}$$

$$f(x) = \begin{cases} |x|+1 & \text{if } x < 0 \\ x^2-1 & \text{if } x \ge 0 \end{cases}$$



$$\lim_{x \to 0^-} f(x) = \boxed{1}$$

$$\lim_{x \to 0^+} f(x) = \boxed{-1}$$

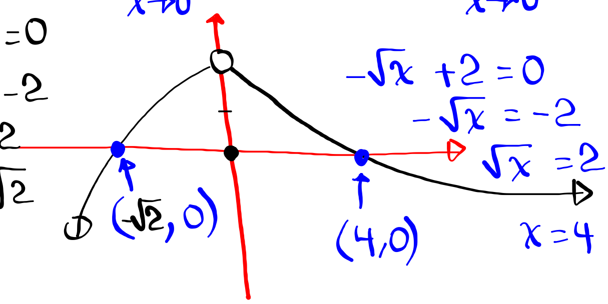
$$\lim_{x \to 0} f(x)$$

$= \text{D.N.E.}$

$$f(x) = \begin{cases} -x^2+2 & \text{if } x<0 \\ 0 & \text{if } x=0 \\ -\sqrt{x}+2 & \text{if } x>0 \end{cases}$$

$\lim_{x\to 0^-} f(x) = \boxed{2}$ $\qquad \lim_{x\to 0^+} f(x) = \boxed{2}$ $\qquad \lim_{x\to 0} f(x) = \boxed{2}$

$-x^2+2=0$
$-x^2=-2$
$x^2=2$
$x=\pm\sqrt{2}$



$-\sqrt{x}+2=0$
$-\sqrt{x}=-2$
$\sqrt{x}=2$
$x=4$

Since

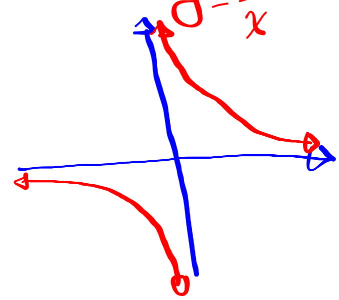
$\lim_{x\to 0} f(x) \neq f(0)$, $f(x)$ is not continuous at $x=0$.

Find $\lim_{x\to\infty} \cos\left(\cancel{\tfrac{1}{x}}^{\to 0}\right)$

$= \cos 0$

$= \boxed{1}$

Hint: Graph $y=\frac{1}{x}$

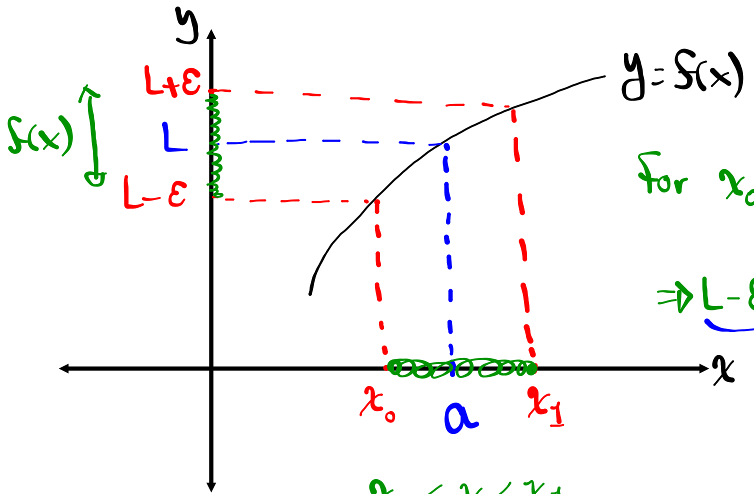


as $x\to\infty$

$\frac{1}{x} \to 0$

$\lim_{x\to 1} \sin\left(\frac{\pi x}{2}\right) = \sin\left(\frac{\pi\cdot 1}{2}\right)$

$= \sin\frac{\pi}{2} = \boxed{1}$



$\varepsilon > 0$

For $x_0 < x < x_1$

$\Rightarrow \underbrace{L-\varepsilon < f(x) < L+\varepsilon}$

$|f(x) - L| < \varepsilon$

epsilon

$x_0 < x < x_1$

$\lim_{x \to a} f(x) = L$

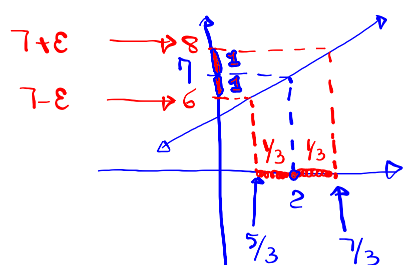
Delta

For any $\varepsilon > 0$, there is always a $\delta > 0$

Such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

Consider $f(x) = 3x + 1$

1) $\lim_{x \to 2} f(x) = 7$



2) Let's say $\varepsilon = 1$

$f(x) = 6 \rightarrow 3x + 1 = 6$

$3x = 5$

$x = 5/3$

$f(x) = 8 \rightarrow 3x + 1 = 8$

$3x = 7$

$x = \frac{7}{3}$

For $\varepsilon = 1 \Rightarrow$ we must be $1/3$ from 2.

$\delta = \frac{1}{3}$

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

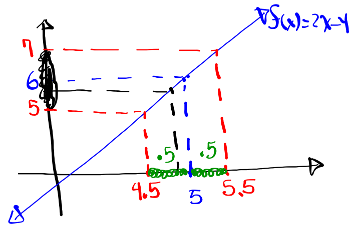
$|f(x) - 7| < 1$ $\qquad |x - 2| < \frac{1}{3}$

February 14, 2022

$f(x) = 2x - 4$

$\lim_{x \to 5} f(x) = 6$ ✓

a, L



How close to 5 should I be, so we will be within 1 unit of 6? .5

$2x-4=5$ $\quad$ $2x-4=7$

$2x=9$ $\quad x=4.5$ $\quad$ $2x=11$ $\quad x=5.5$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|2x-4-6| < \varepsilon$ $\quad$ " $\quad$ $|x-5| < \delta$

$|2x-10| < \varepsilon$

$|2(x-5)| < \varepsilon$

$2|x-5| < \varepsilon$

$|x-5| < \frac{\varepsilon}{2}$

If we let $\delta = \frac{\varepsilon}{2}$

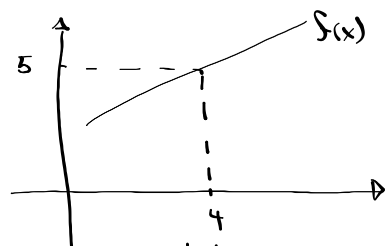
for $\varepsilon = 1 \to \delta = \frac{1}{2}$

for $\varepsilon = 2 \to \delta = 1$

$f(x) = \frac{1}{2}x + 3$

$\lim_{x \to 4} f(x) = 5$

a, L



for $\varepsilon > 0$, there a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$\left|\frac{1}{2}x+3-5\right| < \varepsilon$ $\quad$ " $\quad$ $|x-4| < \delta$

$\left|\frac{1}{2}x - 2\right| < \varepsilon$

$\left|\frac{1}{2}(x-4)\right| < \varepsilon$

$\frac{1}{2}|x-4| < \varepsilon$

$2 \cdot \frac{1}{2}|x-4| < 2\varepsilon$

$|x-4| < 2\varepsilon$

So we let $\delta = 2\varepsilon$

If $\varepsilon = 1 \to \delta = 2$

If $\varepsilon = \frac{1}{2} \to \delta = 1$

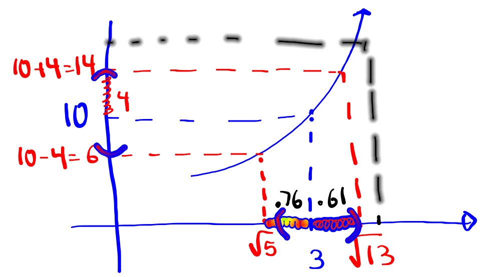
If $\varepsilon = 2 \to \delta = 4$

$f(x) = x^2 + 1$

$\lim_{x \to 3} f(x) = 10$

$x \to 3$ ← a

10 ← L



For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$f(x) = 6$

$x^2 + 1 = 6$

$x^2 = 5 \quad x = \sqrt{5}$

$f(x) = 14$

$x^2 + 1 = 14$

$x^2 = 13 \quad x = \sqrt{13}$

If we wish to be within 4 units of $L = 10$, then we must be within .6 unit of $a = 3$

2.4 3 3.6

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 + 1 - 10| < \varepsilon$ whenever $|x - 3| < \delta$

$|x^2 - 9| < \varepsilon$ " $|x - 3| < \delta$

$|(x+3)(x-3)| < \varepsilon$ " $|x - 3| < \delta$

$|x+3|\,|x-3| < \varepsilon$

Keep

we need to bound it.

$7\,|x-3| < \varepsilon$

$|x-3| < \frac{\varepsilon}{7}$

Pick $\delta = \frac{\varepsilon}{7}$

For $\varepsilon = 4 \Rightarrow \delta = \frac{4}{7} = .6$

For $\varepsilon = 8 \Rightarrow \delta = \frac{8}{7} > 1$ Pick $\delta = 1$

We want δ to be no more than 1.

$|x-3| < 1$

$-1 < x - 3 < 1$

Add 6

$-1+6 < x-3+6 < 1+6$

$5 < x+3 < 7$

$\underbrace{-7 < 5 < x+3 < 7}$

$-7 < x+3 < 7$

$|x+3| < 7$